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A Study of the Histogram of the Normalized Differences Vegetation Index In Terms Of the Statistical Parameters of the Near Infrared and Red Spectral Bands

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Abstract: **In this paper it is studied how the statistical parameters of the individual spectral bands (standard deviations, mean values and correlation coefficient between the near infrared and red band) control the histogram of the Normalized Differences Vegetation Index (NDVI), which is widely used in environmental remote sensing. The basic idea of the whole approach is that as long as the standard deviation of the NDVI values increases, the tonality contrast of the NDVI image improves and the vegetation cover is more clearly mapped. Using proper bivariate Gaussian distributions to describe the histograms of the individual bands, it is shown that the width of the NDVI histogram increases with the standard deviations of the two individual bands and decreases with the correlation coefficient between the bands. It is also shown that the standard deviation of the NDVI presents a weaker decay tendency with the correlation coefficient than the modified simple band ratio does. The results and conclusions of this study may be useful in assessing the performance of the NDVI in environmental research (land cover mapping).**

Keywords: **reflectance, correlation coefficient, spectral band, modified simple vegetation index, normalized differences vegetation index, bivariate distribution.**

I. INTRODUCTION

The simple band ratio u of the reflectance x of the near infrared band to that of the red band (reflectance y) is defined as a function $\varphi(x, y)$ according to the relation:

$$u = x/y = \varphi(x, y) \qquad (1)$$

Reflectances x and y take values between 0 and 1.
In environmental remote sensing, in order to map the vegetation cover of the area under study, the Modified Simple Vegetation Index (MSVI) is used, which is given by [1]:

$$u_{m1} = \arctan(u) \qquad (2)$$

u_{m1} is the value of the MSVI for given reflectances x and y. As long as u_{m1} increases, the density of the vegetation cover is expected to increase.
The Normalized Differences Vegetation Index (NDVI), is the most widely used for land cover mapping and it is defined by [2]:

$$u_{m2} = (x - y)/(x + y) = (u - 1)/(u + 1) \quad (3)$$

u_{m2} is the value of the NDVI for given reflectances x and y. The NDVI value increases with the density of the vegetation cover.

A thorough presentation of the various vegetation indices and their use in environmental research is made by Jensen 2000 [3].

The performance of a vegetation index is often assessed with empirical criteria of response over different vegetation types [4], [5], [6], [7]. This paper focuses on the statistical behavior of the NDVI (histogram width), which is controlled by the statistical parameters of x and y. The basic idea of the whole approach is that as long as the standard deviation of the NDVI values increases, the tonality contrast of the NDVI image improves and the vegetation cover is more clearly mapped. Using proper bivariate Gaussian distributions for x and y bands, an expression for the distribution of NDVI values is derived and its standard deviation is calculated, in terms of the correlation coefficient between x and y. The standard deviation of the NDVI is compared to that of the MSVI, in order to see how the algebraic expression of the vegetation index influences the tonality contrast of the image.

II. DERIVATION OF THE DISTRIBUTIONS OF THE MSVI AND NDVI VALUES

In order to derive expressions for the distributions of the MSVI and NDVI values, a bivariate distribution $f(x, y)$ of the reflectances of bands x and y has to be introduced. The histograms of x and y values may be represented by correlated Gaussian distributions and $f(x, y)$ is given by [8]:

$$f(x,y) = \frac{1}{2\pi\sigma_1\sigma_2} \cdot \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_1)^2}{\sigma_1^2} - 2\rho \cdot \frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2} + \frac{(y-\mu_2)^2}{\sigma_2^2}\right]\right\} \quad (4)$$

σ_1 and μ_1 are the standard deviation and mean value of band x, respectively. σ_2 and μ_2 are the statistical parameters of band y. ρ is the correlation coefficient between x and y bands.

The distribution $g(u)$ of the simple ratio $u = x/y$ is given by [9]:

$$g(u) = \int_0^1 |J| . f(\upsilon, \varphi^{-1}[u, \upsilon]) d\upsilon \quad (5)$$

υ is a variable equal to x, by definition. φ^{-1} is the inverse of function φ which defines the ratio u. $|J|$ is the absolute value of the Jacobian of x and y for u and υ.

Combining relations (1), (4) and (5) gives the following expression for $g(u)$:

$$g(u) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \int_0^1 \frac{x}{u^2} \cdot \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_1)^2}{\sigma_1^2} - 2\rho \cdot \frac{(x-\mu_1)(x/u-\mu_2)}{\sigma_1\sigma_2} + \frac{(x/u-\mu_2)^2}{\sigma_2^2}\right]\right\} dx \quad (6)$$

The expression for the distribution $g_m(u_m)$, of a quantity u_m which is a function of u, may be derived according to the relation [10]:

$$g_m(u_m) = g[F^{-1}(u_m)] \cdot \left|\frac{du}{du_m}\right| \quad (7)$$

F^{-1} is the inverse of function F, according which u_m is defined by u.

Combining relations (2), (6) and (7), the following expression for the distribution $g_{m1}(u_{m1})$ of the MSVI values is derived:

$$g_{m1}(u_{m1}) = \frac{|1-u_{m1}^2|}{2\pi\sigma_1\sigma_2 \tan^2 u_{m1}\sqrt{1-\rho^2}} \int_0^1 x \cdot \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_1)^2}{\sigma_1^2} - 2\rho \cdot \frac{(x-\mu_1)(x/\tan u_{m1}-\mu_2)}{\sigma_1\sigma_2} + \frac{(x/\tan u_{m1}-\mu_2)^2}{\sigma_2^2}\right]\right\} dx \quad (8)$$

Combining (3), (6) and (7) gives the following expression for the distribution $g_{m2}(u_{m2})$ of the NDVI values:

$$g_{m2}(u_{m2}) = \frac{1}{\pi\sigma_1\sigma_2(1+u_{m2})^2\sqrt{1-\rho^2}}\int_0^1 x\cdot\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_1)^2}{\sigma_1^2} - 2\rho\cdot\frac{(x-\mu_1)\cdot[x(1-u_{m2})/(1+u_{m2})-\mu_2]}{\sigma_1\sigma_2} + \frac{[x(1-u_{m2})(1+u_{m2})-\mu_2]^2}{\sigma_2^2}\right]\right\}dx \tag{9}$$

Expressions (8) and (9) may be numerically calculated in order to produce the curves $g_{m1}(u_{m1})$ and $g_{m2}(u_{m2})$, and study the characteristics of the histograms of the MSVI and NDVI.

III. THE NDVI HISTOGRAM AND ITS STANDARD DEVIATION

In Fig. 1 the distribution $g_{m2}(u_{m2})$ of the NDVI values, for various mean values μ_1 and μ_2 of the respective bands x and y, is presented. It can be observed that as long as μ_1 is bigger than μ_2, the histogram of the NDVI values is displaced to the right. This means that the image of the NDVI appears to have bright tones. On the contrary, as long as μ_1 is smaller than μ_2, the histogram is displaced to the left, which means that the NDVI image appears with dark tones.

In Fig. 2 the distribution of the NDVI values for various standard deviations σ_1 and σ_2 of bands x and y is represented. It can be seen that as long as the standard deviations of the spectral bands increase, the width of the NDVI histogram increases. A broad histogram expresses an image with a strong tonality contrast.

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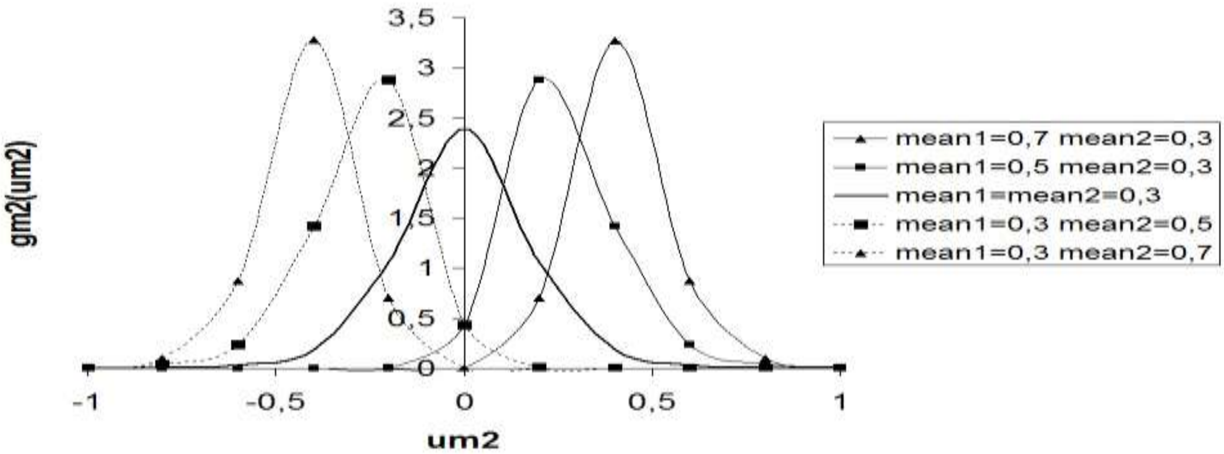

Fig. 1. Distributions of NDVI values for various mean values of x and y. $\rho = 0.5$, $\sigma_1 = \sigma_2 = 0.1$.

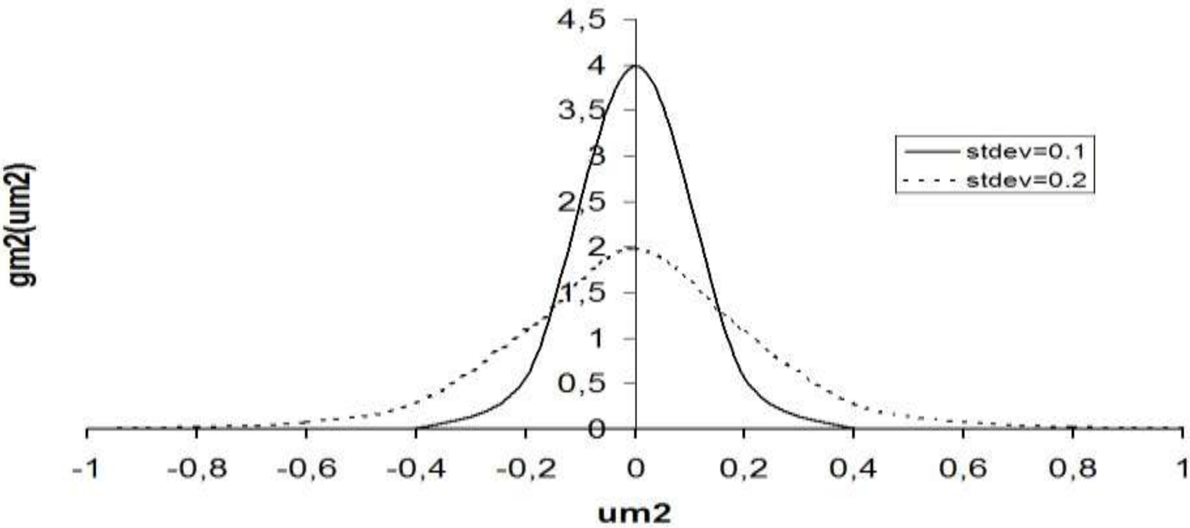

Fig. 2. The distribution of the NDVI values for two different standard deviations of the spectral bands x and y. The standard deviations of x and y are equal. $\rho = 0.5$, $\mu_1 = \mu_2 = 0.5$.

As long as the correlation coefficient ρ increases, the width of the NDVI histogram decreases, as it can be observed in Fig. 3. This means that a strong positive correlation between the bands x and y does not favor a strong tonality contrast of the NDVI image.

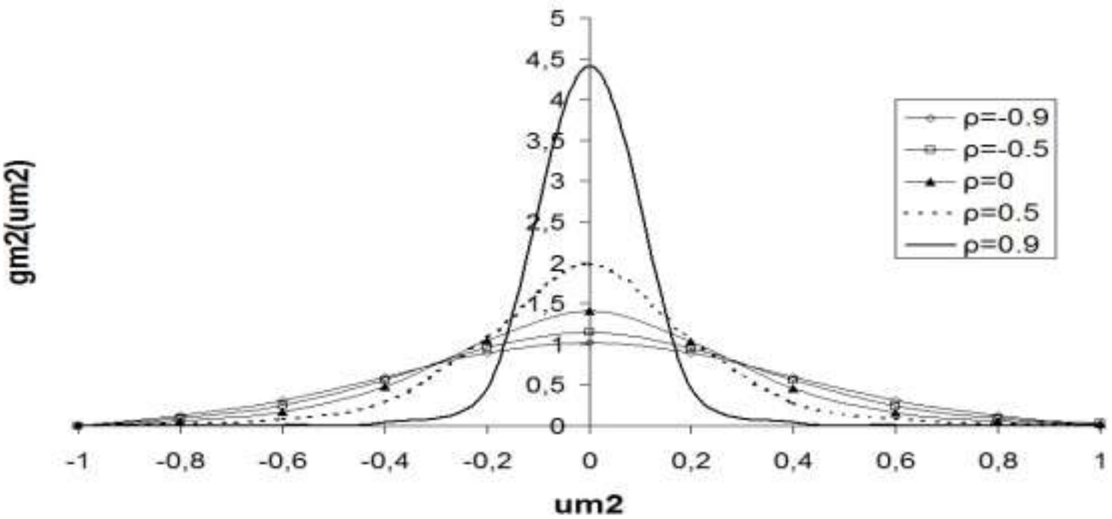

Fig. 3. The distribution of the NDVI values for various correlation coefficients ρ. $\sigma_1 = \sigma_2 = 0.2$, $\mu_1 = \mu_2 = 0.5$.

The statistical behavior of the MSVI is similar to that of the NDVI. It is interesting, however, to compare, in quantitative terms, the widths of the histograms of the two vegetation indices. For such a purpose, the standard deviations of $g_{m1}(u_{m1})$ and $g_{m2}(u_{m2})$ have been numerically computed, and then normalized to a common range of values equal to unity. Since the u_{m1} values of the MSVI belong to the interval between 0 and $\pi/2$, their standard deviations have been divided by 1.57. The u_{m2} values of the NDVI belong to the interval between -1 and 1, therefore their standard deviations have been divided by 2.

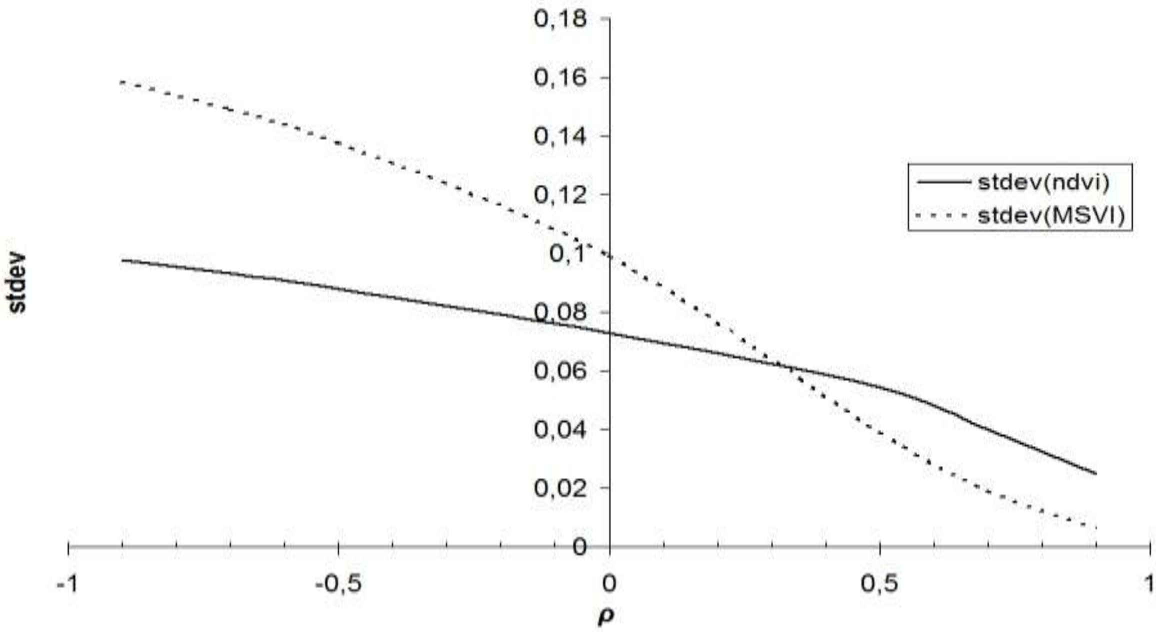

Fig. 4. The variation of the NDVI and MSVI values against the correlation coefficient between the spectral bands. $\sigma_1 = \sigma_2 = 0.1$, $\mu_1 = \mu_2 = 0.5$.

In Fig. 4, the normalized standard deviations of the NDVI and MSVI against the correlation coefficient ρ are presented. The standard deviation of both vegetation indices takes higher values for negatively correlated spectral bands x and y and lower values for positively correlated bands. The decay tendency of the standard deviation of the NDVI with ρ is lesser than that of the MSVI. For strongly and positively correlated spectral bands, with ρ more than 0.5, the normalized standard deviation of the NDVI is greater than that of the MSVI. Therefore, for positively correlated bands, the NDVI image is expected to have a stronger tonality contrast than that of the MSVI image.

IV. CONCLUSIONS

The conclusions of this study may be summarized as following:

The histogram of the NDVI image is controlled by the mean values, standard deviations and correlation coefficient between the spectral bands.

As long as the mean value of band *x* is greater than that of *y*, the NDVI histogram tends to be displaced to the right and take high NDVI values.

As long as the standard deviation of the spectral bands increases, the width and standard deviation of the NDVI histogram also increases.

A negative correlation coefficient between the spectral bands *x* and *y*, favours the production of a wide NDVI histogram with a relatively large standard deviation.

As long as the correlation coefficient increases, the standard deviation of the NDVI image decreases.

Compared to the statistical behaviour of the MSVI, the standard deviation of the NDVI presents a weaker decay tendency with the correlation coefficient. For non correlated or negatively correlated spectral zones, the standard deviation of the MSVI is higher than that of the NDVI. On the contrary, the standard deviation of the NDVI is higher than that of the MSVI, for a strong positive correlation between the bands. In such a case, the NDVI image is expected to have a stronger tonality contrast than the MSVI image.

The results and conclusions of this paper may be useful in environmental research (mapping the vegetation cover by satellite imagery).

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